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STATE FEEDBACK CONTROL OF OUPLED-TANK LEVEL SYSTEM BASED ON POLE-PLACEMENT

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ABSTRACT

A state feedback control strategy for Coupled-tank level system based on Pole-Placement is presented in this paper. First, the operation point is determined when given the desired the controlled level of the lower tank according to the established nonlinear differential equations of coupled-tank system. Then, the linear state space model is obtained by using Taylor first-order expansion method and state definitions. Finally, the gain matrix of the state feedback controller is derived by placing the poles of the closed-loop system at the desired placements. The simulation results show that the designed closed-loop control system has better dynamic and steady-state performance.

KEYWORDS

Coupled-tank, state feedback, Pole-Placement, linearization.

1. INTRODUCTION

The Coupled-tank level system is a single-input, single-output, nonlinear experimental system which can simulate a lot of tank flow system in many process industries such as petrochemical industries, paper making industries, water treatment industries, etc. As one popular experimental system in laboratories, the coupled-tank system can be used for investigating or verifying various control methods, such as PI control, variable structure control, auto-disturbance rejection control, adaptive control, and fuzzy control etc [1,2]. For some controllers, their complicated structures, fussy parameter turning, or large real-time calculations, make them difficult to implement in real system. This paper proposes a state feedback control strategy based on pole-Placement for Coupled-tank system. The designed state feedback controller has simple structure. Using the state feedback method can achieve arbitrary configuration of the pole and it is easy to implement. First, the nonlinear system equation established by the mechanism is given, and the operation point can be calculated according to the nonlinear equation. Then, using Taylor series expansion and defining the states, linear state space equations is obtained, the state feedback gain matrix is derived by pole-Placement method. Finally, the simulation results with the closed-loop verify the feasibility and effectiveness of the control strategy proposed in this paper [3].

The paper is organized as follows. In Section two, notation the coupled-tank level system and its mathematical model is introduced. The linearization process of the system and the derivation of the state space equation are described in Section four. The State feedback controller design gives process is given in Section six, and results and discussion are presented in Section seven. Finally, in Section eight the main conclusions are drawn.

2. COUPLED-TANK LEVEL SYSTEM AND MATHEMATICAL MODEL

A typical Coupled-tank level system is shown in Figure 1. The system consists of a bottom sink and two upper and lower cylindrical tanks with the same cross-sectional area. An adjustable flow pump pumps water from the bottom sink into the upper tank. The water in the upper tank flows into the lower tank through the outlet at the bottom, and the water in the tank also flows into the sink through the bottom outlet. The purpose of the Coupled-tank level system is control the level of the lower tank by adjusting the pump flow [4].

Figure 1: Coupled-tank level system structure diagram

According to conservation principle on Total Mass Balance and well-known Bernulli's principle, the mathematical model of a Coupled-tank can be described as [5]:

$$A_1 \frac{dh_1}{dt} = Q - \alpha_1 A_{01}\sqrt{2gh_1} = f_1 \tag{1a}$$

$$A_2 \frac{dh_2}{dt} = \alpha_1 A_{01}\sqrt{2gh_1} - \alpha_2 A_{02}\sqrt{2gh_2} = f_2 \tag{1b}$$

here, h_1 and h_2 are the upper and lower tank levels respectively, A_{t1}=A_{t2}=15.5179 cm^2 are the cross-sectional area of the two tanks, A_{o1}=A_{o2}=0.1781cm^2 is the cross-sectional area of the outlet hole at the bottom of the tank, g=981cm/s^2 is the gravity acceleration, $Q = K_p V_p$ is the flow rate of the water pumped into the upper tank, V_p is Pump drive voltage, and K_p= 3.3 is the correlation coefficient.

3. PARAMETER IDENTIFICATION

Coupled-tank mathematic model is established in an ideal environment. But in the actual experiment process, due to the effect of level height and pressure, it cause water flow through the outlet section when the volume shrinks, Effective cross section is less than A_{o1} and A_{o2}, this has error compared to the actual equipment. The accuracy of the digital simulation results is closely related to the accuracy of the models and parameters used. In order to make the experiment more accurate, the parameter identification method can be used to correction the mathematical model of the Coupled-tank. Parameter identification, it is to combine the Coupled-tank mathematic model with experimental data, make the digital results calculated by the mathematical model best fit the test data.

Since the actual outlet flow is less than the theoretical flow, two parameters can be added at the outlet of the tank model α_1 α_2, through constant modify α_1 and α_2 value, make the mathematical model and test data best fit. After repeated experiments, when α_1=0.989, α_2=0.93. The best fit between Coupled-tank mathematical model liquid level and actual liquid level. is shown in Figure 2.

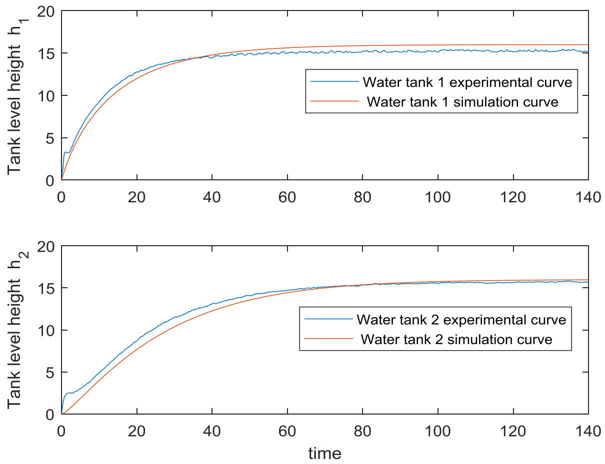

Figure 2: Coupled-tank parameter identification liquid level curve

4. CALCULATION OF STEADY-STATE OPERATING POINT

Steady-state operating point of a model, also called an equilibrium or trim condition, includes state variables that do not change with time. At this time, the derivative of all state variables in the system is 0, and the system is in a stable working state [6].

From the mathematical model of Coupled-tank system (1), it can be seen that the system is a typical nonlinear system. In order to obtain the linear state space equation and use the state space analysis method to design closed-loop control system, we need to linearize the nonlinear equation. First, the operating point the nonlinear system must be calculated before linearization. Let the equation (1a) and (1b) equal to 0

$$K_p V_p - \alpha_1 A_{o1}\sqrt{gh_1} = 0$$

$$\alpha_1 A_{o1}\sqrt{2gh_1} - \alpha_2 A_{o2}\sqrt{2gh_2} = 0$$

and set the h_2 as desired constant value h_{2s}, then we can derive the only solution of two variables of V_p h_1 as V_{ps} h_{1s} by solving two equation, the pair (V_p h_1 as V_{ps} h_{1s}) is then an operating point of this nonlinear system.

5. Tank model linearization and state space model

The control of inlet flow in Coupled-tank directly affects the dynamic balance of the system. The output and state of the system is moved under the constraints of the tank system. So when the system moves to equilibrium. It can control the movement status and output of the system. Before that, the tank model should first be linearized at the equilibrium position.

Assuming the system run near an operating point and the deviation at the operating point is small, we can linearize the system (1) using the Taylor series expansion method. the linear state space equation can be described by the following equation

$$\begin{cases} \dfrac{d\,(h_1 - h_{1s})}{dt} = \dfrac{\partial f_1(h_1,h_2,V_p)}{\partial h_1}\bigg|_{h_{1s},h_{2s},V_{ps}}(h_1 - h_{1s}) + \dfrac{\partial f_1(h_1,h_2,V_p)}{\partial V_p}\bigg|_{h_{1s},h_{2s},V_{ps}}(V_p - V_{ps}) \\ \quad = -\dfrac{A_{o1}g}{A_{t1}\sqrt{2gh_1}}(h_1 - h_{2s}) + \dfrac{K_p}{A_{t1}}(V_p - V_{ps}) \\ \dfrac{d\,(h_2 - h_s)}{dt} = \dfrac{\partial f_2(h_1,h_2,V_p)}{\partial h_1}\bigg|_{h_{1s},h_{2s},V_{ps}}(h_1 - h_{1s}) + \dfrac{\partial f_2(h_1,h_2,V_p)}{\partial h_2}\bigg|_{h_{1s},h_{2s},V_{p}}(h_2 - h_{2s}) \\ \quad = \dfrac{A_o g}{A_{t2}\sqrt{2gh_1}}(h_1 - h_1) - \dfrac{A_{o2}g}{A_{t2}\sqrt{2gh_2}}(h_2 - h_{2s}) \end{cases} \qquad (2)$$

Expressing the system state x x_2 inputs u as $x_1 = h_1 - h_{1s}$ $x_2 = h_2$ h_{2s} $u = V_p$ V_{ps} $\mathbf{x} = [x_1, x\ ]^T$, which also represent the deviation from the operating point, The state space expression of the Coupled-tank around the operating point is

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

Take linear state space equation (2) to state space expression(3)

$$\frac{d\mathbf{x}}{dt} = \begin{vmatrix} -\dfrac{A_{o1}g}{A_{t1}\sqrt{2gh_1}} & 0 \\ \dfrac{A_{o1}g}{A_{t2}\sqrt{2gh_1}} & -\dfrac{A_{o1}g}{A_{t2}\sqrt{2gh_1}} \end{vmatrix}\mathbf{x} + \begin{bmatrix} \dfrac{k_p}{A_{t1}} \\ 0 \end{bmatrix} u$$

Now, we choose a desired level of the lower tank h_{2s} =15, the operating point (V_{ps} =9.26, h_{1s} =15 h_{2s} =15) can be derived.

$$A = \begin{vmatrix} -0.0656 & 0 \\ 0.0656 & -0.0656 \end{vmatrix},\ B = \begin{bmatrix} 0.213 \\ 0 \end{bmatrix},\ C = [0 \quad 1],\ D = 0.$$

6. STATE FEEDBACK CONTROLLER DESIGN

The dynamic performance of a linear system, such as system stability, overshoot in the time domain analysis, and transition time, depends primarily on the pole position of the system. The general method of pole-placement can be estimated by scaling (such as the root-locus method) and empirically to determine specifically [7]. Placing the closed-loop pole set to the desired position is equivalent to achieving the desired dynamic performance of the integrated linear system. Based on the state space model (3), this paper uses the pole-placement method to determine the state feedback controller gain matrix. Before the pole-placement, first, we must determine the controllability of the system. If the system is fully controllable, we can place the poles arbitrarily [8]. The control matrix of the Coupled-tank level system is:

$$B = \begin{bmatrix} 0.213 \\ 0 \end{bmatrix},\ AB = \begin{bmatrix} 0.066 & 0 \\ 0.066 & -0.066 \end{bmatrix}\begin{bmatrix} 0.213 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.014 \\ 0.014 \end{bmatrix}$$

So the controllability discriminant matrix:

$$Q_c \quad [B \quad AB] = \begin{bmatrix} 0.213 & -0.014 \\ 0 & 0.014 \end{bmatrix}$$

because $\mathrm{rank}(Q_c) = 2$, Therefore, any configuration of poles can be performed.

Setting the state feedback matrix K be:

$$K = [k_1 \quad k_2]$$

After introducing state feedback, the characteristic polynomial of the closed-loop system is:

$$\det(sI - A + BK) = \det\left\{\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -0.066 & 0 \\ 0.066 & -0.066 \end{bmatrix} + \begin{bmatrix} 0.213 \\ 0 \end{bmatrix}[k_1 \quad k_2]\right\}$$

The result is

$$\begin{vmatrix} s+0.066+0.213k_1 & 0.213k_2 \\ -0.066 & s+0.066 \end{vmatrix} = s^2 + (0.132+0.213k_1)s + 0.014k_1 + 0.014k_2 + 0.0044$$

Now configure the system's closed-loop pole to:

$$s_1 = -1, s_2 = -2$$

So the desired characteristic polynomial is:

$$(s+1)(s+2) = s^2 + 3s + 2$$

According to the corresponding coefficient is equal:

$$0.132 + 0.213k_1 = 0.014k_1 + 0.014k_2 + 0.0044 = 2$$

Can be solved

$$k_1 = 13.46, k_2 = 129.08$$

The resulting state feedback controller gain matrix K is:

$$K = [13.46 \quad 129.08]$$

7. SIMULATION RESULTS

Based on the SIMULINK established Coupled-tank level feedback control simulation system shown in Figure 3.

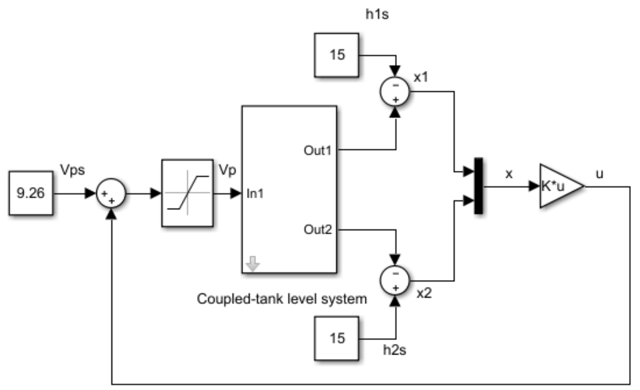

Figure 3: Status feedback control simulation system for Coupled-tank

Assume that the initial level of the system $h_1(0) = 0\text{cm}$, $h_2(0) = 0\text{cm}$, operating system and the set value $h_{2s} = 15\text{cm}$. The simulation and experimental results are shown in Figure 4.

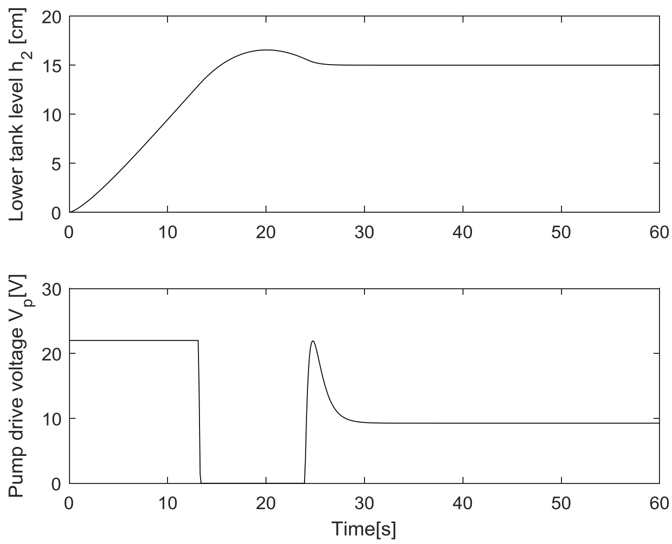

Figure 4: Closed-loop control system simulation results

From the simulation results, the state feedback control has a good effect on the closed loop, Compared to PID control, this control method is simple and saves a lot of debugging time. lower tank level reaches the steady state equilibrium quickly, and the overshoot is small. It can be seen that the closed-loop system has ideal dynamic and steady-state performance.

8. CONCLUSION

For the Coupled-tank level system, this paper introduces a design method of state feedback controller. Firstly, according to the obtained mechanism model, Obtain a state space model by linearization, and then according to the pole placement method obtain the gain matrix of the state feedback controller. Compared with other control methods, the controller only requires the multiplication and addition operations in the real system implementation process, which has strong real-time performance and is easy to implement.

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