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VARIABLE STRUCTURE PID CONTROL OF FOUR-TANK SYSTEM WITH INPUT CONSTRAINT

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ABSTRACT

The four-tank system has a wide range of application backgrounds in process control, and usually has such prominent features as slow, time-delay, coupling, multiple-input, multiple-output, and strong nonlinearity, which imposes higher requirements on control strategies. In this paper, considering the actually existing input constraint, a variable structure PID control method is studied based on the feedback linearization of nonlinear system. Simulation results show that this control method improves the control performance obviously, compared with conventional PID control and is easier to apply in engineering.

KEYWORDS

Four-tank system, Input constraint, Feedback linearization, Variable structure PID control.

1. INTRODUCTION

Since the invention of the PID control algorithm, it has been widely applied to the automatic control of various systems with its simple structure, easy implementation, and strong robustness. At present, PID control is still the dominant control method and has obtained in-depth research. However, the conventional PID control algorithm cannot handle the control quantity constraint, and can only use the high or low selector to relatively avoid the saturation problem of the actuator. The design process is very tedious and is not conducive to the operator's readjustment. For the control system, the constraints are ubiquitous. If the processing is not done properly, it often causes the degradation of the system control engineering, and even leads to system instability. In order to solve the constraint system control problem, many scholars have conducted extensive research and proposed many effective algorithms. Scottedward proposed a variable structure PID algorithm that not only preserves the simplicity of conventional PID control algorithms, but also improves the control performance of the system [1]. However, this algorithm is only applicable to single-input single-output systems. In this paper, a four-capacity water tank system is taken as an example. For a multiple-input multi-output system with nonlinear characteristics, it is decoupled by feedback linearization and combined with a variable structure PID control algorithm. The simulation results are given.

2. MATHEMATICAL MODEL OF FOUR-TANK SYSTEM

The physical model of the four-tank water tank system is shown in Figure 1. Four interrelated water tanks with uniform cross-section and two pumps with water basins form an autonomous closed and recirculating system. Pump 1 pumps water from the reservoir to tank 1 (Tank 1) and its diagonal tank 4 (Tank 4); Pump 2 pumps water from the reservoir to tank 2 (Tank 2) And its diagonal position of the high tank 3 (Tank3). Tank 1 and tank 2 level can be measured, the manipulated variable for the speed of the two pumps, the input voltage control [2].

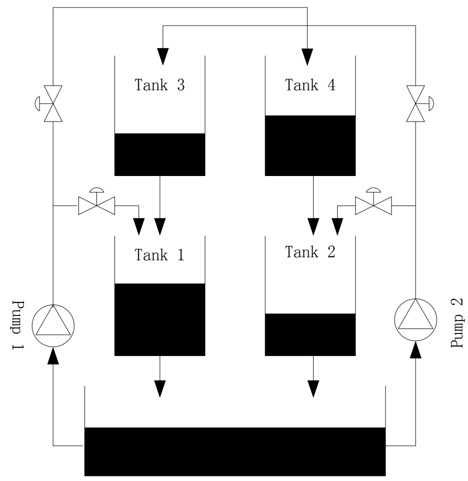

Figure 1: Schematic structure of four-tank system

According to the law of conservation of mass, apply

$$A\frac{dh}{dt} = q_{in} \quad q_{out}$$

to each tank, we get

$$A_1\frac{dh_1}{dt} = -a_1\sqrt{2gh_1} + a_3\sqrt{2gh_3} + \gamma_1 k_1 v_1$$

$$A_2\frac{dh_2}{dt} = -a_2\sqrt{2gh_2} + a_4\sqrt{2gh_4} + \gamma_2 k_2 v_2$$

$$A_3\frac{dh_3}{dt} = -a_3\sqrt{2gh_3} + (1-\gamma_2) k_2 v_2$$

$$A_4\frac{dh_4}{dt} = -a_4\sqrt{2gh_4} + (1-\gamma_1) k_1 v_1$$

in which $h_i (i=1,2,3,4)$ is the height of the liquid level in tank i; $A_i (i=1,2,3,4)$ is the cross-sectional area of the tanks; a_i is the outlet area of the tank i; V_1, V_2 is the adjustable input of the system, namely the rotational speed of pump 1 and pump 2; γ_1, γ_2 is the shunt proportional coefficient.

3. NONLINEAR SYSTEM FEEDBACK LINEARIZATION

Considering Time-varying nonlinear systems

$$\begin{cases} \dot{x}(t) = f(x,t) + \sum_{i=1}^{m} g_i(x,t) u_i(t) \\ y(t) = h(x,t) \end{cases} \tag{1}$$

in which, $x \in R^n$ is a state vector; $u \in R^m$ is a control input; $y \in R^p$ is a controlled input; respectively, $f(x,t), g(x,t), h(x,t)$ a smooth vector field with a corresponding dimension. The precondition for achieving decoupling at the same time as linearization is m=p, that is, the input dimension is the same as the output dimension.
The problem of feedback linearization and decoupling of this nonlinear system is to find the state feedback

$$u = F(x,t) + G(x,t)\nu(t) \tag{2}$$

Therefore, a linear system equivalent to the original system and each input component only affects the corresponding output component is established, and the decoupling system dynamics can be adjusted by several configuration methods, where $F(x,t) \in R^{m\times 1}$, $G(x,t) \in R^{m\times m}$, $\nu(t) \in R^m$ are new references enter.

Outputs y_i to each component of the multiple-input multiple-output system for a derivative of time t until u appears in the expression.

$$d_{y_j}/d_t = d/d_t[h_j(x,t)] = \partial/\partial t[h_j(x,t)] + \partial/\partial x[h_j(x,t)]\dot{x}(t) \tag{3}$$

Substituting (1)

$$\begin{aligned} d_{y_j}/d_t &= d/d_t[h_j(x,t)] = \partial/\partial t[h_j(x,t)] + \partial/\partial x[h_j(x,t)]\{f(x,t) + g(x,t)u(t)\} \\ &= \partial/\partial t[h_j(x,t)] + \partial/\partial x[h_j(x,t)] f(x,t) + \partial/\partial x[h_j(x,t)] g(x,t)u(t) \end{aligned} \tag{4}$$

For the sake of simplicity, it will be recorded

$$L_f h_j = \partial/\partial x[h_j(x,t)] f(x,t) \tag{5}$$

That is, the derivative of h_j with respect to f is the same as the derivative of f.

$$L_g h_j = \partial/\partial x[h_j(x,t)] g(x,t) \tag{6}$$

If $L_g h_j = 0$, that is, the control input u does not appear in d_{y_j}/d_t, then the output component needs to be further derived. Note $L_f^0 h_j = h_j$ that if r_j is the smallest integer (ie relative order) in which at least one control input component appears in $y_j^{(r_j)} \cong d^{r_j} y_j / dt^{r_j}$, then $y_j^{(r_j)} = L_f^{r_j} h_j + L_g^{r_j} h_j u$.

Definition

$$\begin{aligned} Y(t) &= [y_1^{(r_1)}, \ldots, y_m^{(r_m)}]^T \\ L(x,t) &= [L_f^{r_1} h_1(x,t), \ldots, L_f^{r_m} h_m(x,t)]^T \end{aligned} \tag{7}$$

That is $Y(t) = L(x,t) + E(x,t)u$, matrix $E(x,t) \in R^{m\times m}$. The definition is as follows

$$E(x,t) = \begin{bmatrix} L_{g1} L_f^{r_1-1} h_1 & \ldots & L_{gm} L_f^{r_1-1} h_1 \\ \ldots & & \ldots \\ L_{g1} L_f^{r_m-1} h_m & \ldots & L_{gm} L_f^{r_m-1} h_m \end{bmatrix} \tag{8}$$

If $E(x,t)$ is non-singular in the domain Ω of the point X_0, it is transformed by the input

$$u(t) = -E^{-1}(x,t)L(x,t) + E^{-1}(x,t)\nu(t) \tag{9}$$

A linear system that is equivalent to the original system and whose input components only affect the corresponding output components can be established $Y(t) = \nu(t)$.

In order to make the components of the new input $\nu(t)$ only affect the dynamic characteristics of the corresponding decoupling subsystem, redefine the input transform

$$u(t) = -E^{-1}(x,t)L(x,t) - E^{-1}(x,t)M(x,t) + E^{-1}(x,t)P\nu(t) \tag{10}$$

In the formula

$$M_i(x,t) = \begin{cases} 0, r_i = 0 \\ \sum_{k=0}^{r_i-1} m_{ki} L_f^h h(x), r_i \neq 0 \end{cases}$$, P=diag $\{pi\}$, i=1,2,...,m,

The m_{ki}, p_i (11)
is an optional adjustment parameter. The above formula can be further transformed into

$$Y(t) = -M(x,t) + P\nu(t) \tag{12}$$

Then the i-th equivalent decoupled linear subsystem of equation (1) can be represented as

$$y_i^{(r_i)}(t) + m_{(r_i-1)i} y_i^{(r_i-1)}(t) + \ldots + m_{0i} y_i(t) = p_i \upsilon_i(t) \tag{13}$$

At this point, each subsystem appears as a single-input single-output linear r_i-order ordinary differential equation. Defining the state variable of the four-capacity system (1) is $x = [h_1, h_2, h_3, h_4]^T$, The controlled input is $u = [\nu_1, \nu_2]^T$, the controlled output is $y = [h_1, h_2]^T$, according to the above decoupling algorithm, from

$$\frac{dy_1}{dt} = \frac{-a_1\sqrt{2gh_1} + a_3\sqrt{2gh} \quad \gamma_1 k_1 u_1}{A_1}$$

$$\frac{dy_2}{dt} = \frac{-a_2\sqrt{2gh_2} + a_4\sqrt{2gh_4} + \gamma_2 k_2 u_2}{A_2}.$$

It can be seen that $r_1 = r_2 = 1$, and further calculation leads to

$$L(x,t) = \begin{bmatrix} \dfrac{-a_1\sqrt{2gh_1} + a_3\sqrt{2gh_3}}{A_1} \\ \dfrac{a_2\sqrt{2gh_2} + a_4\sqrt{2gh_4}}{A_2} \end{bmatrix}, E(x,t) = \begin{bmatrix} \dfrac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \dfrac{\gamma_2 k_2}{A_2} \end{bmatrix},$$

It can be seen that E is non-singular so that the four-capacity system can be decoupled.

4. VARIABLE STRUCTURE PID CONTROL

The structure of variable structure PID controller is shown in Figure 2 [1]. The algorithm is

$$\dot{\eta} = \begin{cases} -\alpha(u_n - u_s)/K_1, u_n \neq u_s \text{ and } e(u_n - \bar{u}) > 0 \\ e, \text{others} \end{cases} \tag{14}$$

In the formula, $\bar{u} \cong (u_{\min} + u_{\max})/2$, α is the adjustment parameter, please indicate the convergence speed of the control quantity back to the restriction range.

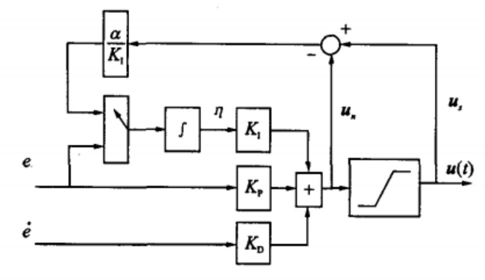

Figure 2: Variable Structure PID Control

The algorithm has the following features:

(1) The designer can make full use of the existing PID knowledge for parameter adjustment. If saturation does not occur, the behavior of the closed-loop system and the linear analysis result are the same;
(2) Intuitively, using the variable structure PID design parameter α can easily adjust the closed-loop performance index. According to experience, when the system is operating in the saturation region, the integrator loop adjustment time should be 2 to 5 times faster than the design of the closed-loop system.
(3) Due to the switching behavior of the variable structure PID method and the feedback of saturation error $u_n - u_s$, when u_n is not equal to u_s , the nominal control amount u_n tends to remain relatively close to the actuator saturation limit, and therefore, the variable structure PID controller returns to the linear range. The speed is faster than the normal saturation method [3].

5. SIMULATION RESULT

As a simulation example, physical parameters of four-tank system are given by Johanson [4]. The conventional PID settings are given by R. Suja Mani Malar [5]. Set $\alpha = 0.1$. Figures 3 to 6 show the simulation results obtained by the conventional PID algorithm and variable structure PID algorithm after decoupling the system. From the figure, we can see that in the process of the steady state of the liquid level (h1 and h2), compared with the conventional PID control, the response of the liquid level curve under the variable structure PID control algorithm is faster, and almost no overshoot; The overshoot of the liquid level %12 around the conventional PID control is prone to overflow when the target liquid level is high, which is very dangerous in industrial production involving toxic and flammable substances [6-10]. From the output of the two pumps, it can be observed that since the variable structure PID algorithm can actively adjust the control amount when the saturation occurs, the control input can be returned to the constraint as soon as possible, so the variable structure PID control returns to the control range within a constrained range [11-13]. The PID control is much faster, while the conventional PID control can only passively adjust the control input back to the constrained range as the deviation decreases. The variable structure PID control has practical significance for saving energy, achieving optimal control, and improving economic efficiency. The conventional PID control is kept in saturation for a long time, so that the controller's regulating effect is greatly reduced.

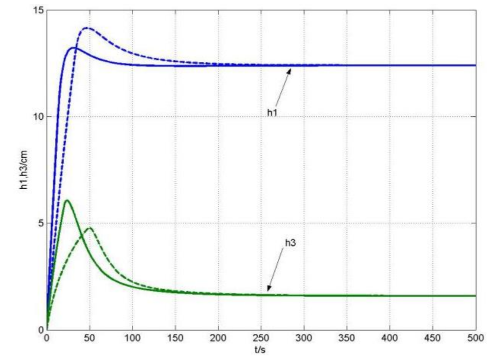

Figure 3: Response curves of level h_1 and h_3

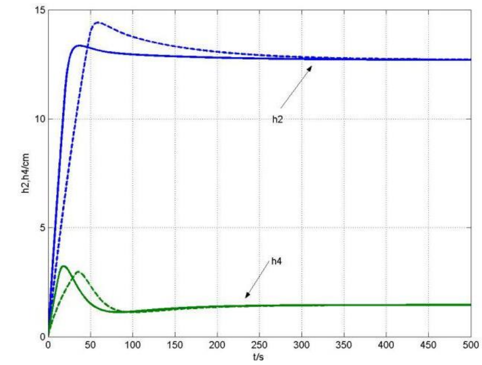

Figure 4: Response curves of level h_2 and h_4

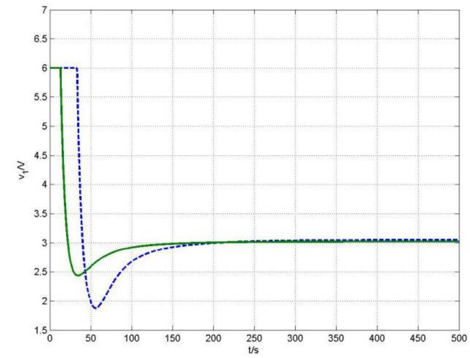

Figure 5: Response curve of v_1

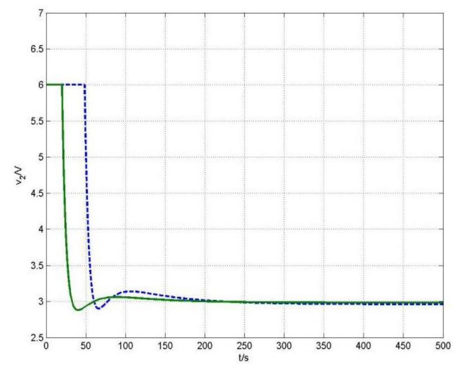

Figure 6: Response curve of v_2

6. CONCLUSION

In this paper, based on feedback linearization of nonlinear systems and its static decoupling, a variable structure PID control method is introduced. This method has the advantages of improving the robust performance of the system with control constraints, simple structure and easy parameter adjustment. Taking the four systems as the control object, the experimental results of the rapid prototype simulation of the variable structure PID control are given. The comparison with the conventional PID control results shows the superiority of this method.

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